TECHNICAL NOTE

The effect of channeling on heat transfer across a horizontal layer of a porous medium

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Kazmierczak and Muley (1994) have reported the results of experiments on heat transfer across a horizontal layer of a porous medium consisting of a matrix of spherical glass beads saturated with water. They observed that the presence of a thin layer of clear fluid just below the rigid top boundary led to a substantial increase in heat transfer. They did not attempt a quantitative comparison with the available theoretical material. This is done in the present note. The comparison is not straightforward because the published papers involving numerical calculations of heat transfer (Catton 1985; Poulikakos et al. 1986; Poulikakos 1985) have reported results for parameter values that are not pertinent to the latest experiments. An indirect comparison, however, is possible.

Figure 4 of Kazmierczak and Muley (1994) indicates that in the range of Rayleigh number Ra from 200 to 1000 the Nusselt number Nu is approximately a linear function Ra, both in the absence and presence of the clear layer, in accordance with the approximate theoretical relationship that is expected to hold, namely,

$$\mathrm{Nu} = \mathrm{Ra}/\mathrm{Ra}_c$$

where Ra_c is the critical value of Ra. With the clear layer present, a given value of the Nusselt number is attained at a Ra value that is about 0.75 of the corresponding value with the clear layer absent. This suggests that the clear layer has the effect of lowering Ra_c by about 25 percent.

The literature reviews by Prasad (1991) and Nield and Bejan (1992) contain references to a number of papers in which the value of Ra_c is predicted, but again many of these treat configurations and parameter ranges that are different from those in the latest experiments. In these, the depth ratio $d_m/d_f = 30$ (so $d_f/d_m = 0.03$), the Darcy number $\delta^2 = 3.96 \times 10^{-7}$ (so $\delta = 6.3 \times 10^{-4}$), the thermal conductivity ratio $k_m/k_f = 1.5$, and $\Delta = 0.2$. The conductivity ratio has been estimated from the formula $k_m = \phi k_f + (1 - \phi)k_s$, where the porosity $\phi = 0.37$. Here the subscripts m, f, and s refer to the medium, fluid, and solid, respectively. In terms of the permeability K and the Beavers–Joseph constant α (taken as 0.1), $\delta = K^{1/2}/d_m$ and $\Delta = K^{1/2}/\alpha d_f$, where here $d_f = 0.004$ m, $d_m = 0.123$ m and $K = 6.38 \times 10^{-9}$ m^2. Estimates based on the analytical formulas given by Nield (1977, 1983) and Pillatsis et al. (1987) indicate that, for the case of a thin fluid layer, Ra_c does not depend strongly on k_m/k_f (when that is close to unity) nor on Δ; rather, the important parameters are d_m/d_f and δ.

Chen and Chen (1988) studied the configuration of interest, that of a fluid layer overlying a porous medium layer, with rigid conducting boundaries. They performed calculations for the case $k_f/k_m = 0.7$ and $\delta = 0.002$. They reported the following results for the pair (d_f/d_m, Ra_c): (0.001, 39.422), (0.01, 36.702), and (0.04, 24.772). This indicates that for $d_f/d_m = 0.03$ the predicted reduction in Ra_c is about 30 percent. In the latest experiments, the value of δ is about three times smaller, and one would anticipate a larger reduction in Ra_c. Results for a range of values of δ are reported by Taslim and Narusawa (1989). Their Figure 2 is here reproduced as Figure 1. These results pertain to a porous layer of depth $2d_m$ sandwiched between two fluid layers, each of depth d_f, the whole sandwich lying between rigid conducting boundaries. The figure shows that as d_m/d_f varies, the Ra_{mc}-versus-δ curve more or less retains its shape but is displaced horizontally. Measurements made on the curves indicate that for a given value of Ra_{mc}, δ varies with d_m/d_f in a multiplicative manner (their logarithms are linearly

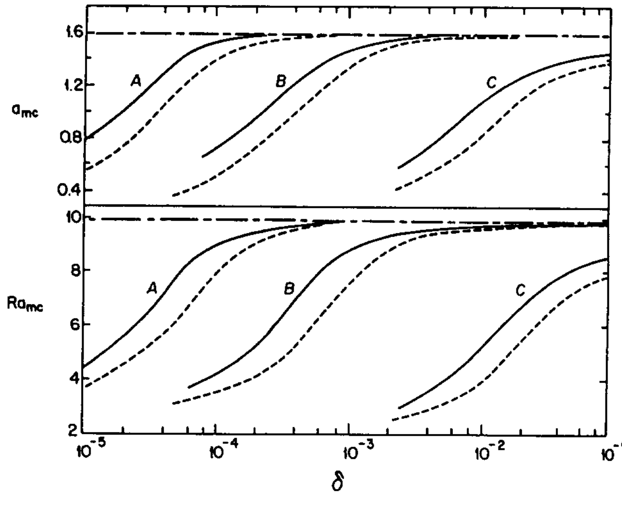

Figure 1 (Reproduction of Figure 2 of Taslim and Narusawa (1989)). Variation of the critical Rayleigh–Darcy number Ra_{mc} and critical wavenumber a_{mc} with δ, for the case of a porous medium layer of depth $2d_m$ sandwiched between two fluid layers, each of depth d_f. The horizontal broken line indicates the porous layer limit ($\mathrm{Ra}_{mc} = \pi^2 = 9.8696$, $a_{mc} = \pi/2 = 1.5708$). The continuous curves correspond to rigid top and bottom boundaries; the slashed curves correspond to free top and bottom boundaries (A) $d_m/d_f = 500$; (B) $d_m/d_f = 100$; (C) $d_m/d_f = 10$

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related): if d_m/d_f is increased by a factor of 10, then δ is decreased by a factor of 45. This means that if d_m/d_f is increased from 30 to 100, then the corresponding δ is decreased by a factor of 6.7.

We are interested in the case $d_m/d_f = 30$, $\delta = 6.3 \times 10^{-4}$. The value of Ra_{mc} for this case is about the same as that for $d_m/d_f = 100$, $\delta = 9.4 \times 10^{-5}$, and reading from the solid curve B we find that $Ra_{mc} = 4$ approximately. In the absence of the two clear fluid layers, $Ra_{mc} = \pi^2 = 9.8696$. The two clear fluid layers cause Ra_{mc} to be reduced by 60 percent. It is plausible that a single clear fluid layer would reduce the critical Rayleigh number by 30 percent. This is just a little larger than the 25 percent that we estimated from the observed heat transfer results of Kazmierczak and Muley (1994).

There is a further consideration. Kazmierczak and Muley (1994) eliminated the clear fluid layer in their later experiments by adding more glass beads. This addition effectively increased the depth of the porous medium from about 0.123 m to 0.127 m. The Ra_{mc} as defined by Taslim and Narusawa (1989) is proportional to the depth of the porous layer. This means that for a proper comparison, the Ra values for later experiments of Kazmierczak and Muley (1994) should be decreased by about 3 percent. This has the effect of reducing the observed change in heat transfer by that amount. And there is one more complication. If the clear fluid layer is a consequence of settling of the glass spheres, the effective porosity, and hence the permeability calculated from the Ergun form of the Kozeny equation, and hence the effective Rayleigh number, needs to be adjusted. This complication does not significantly affect the above argument, which is primarily based on the ratio of Rayleigh numbers, but it would affect a precise comparison because K enters into the parameters δ and Δ as well as Ra.

In summary, the magnitude of the observed change in heat transfer observed by Kazmierczak and Muley (1994) is consistent with the predictions of the available theoretical work. Further experiments, extending the Rayleigh number range down to the critical value, would be welcome. So would new calculations made explicitly for the parameter values pertaining to the experiments.

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